CHAPTER 9

Study Guide and Review

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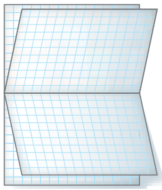
GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Exponential Functions (Lesson 9-1)

- An exponential function is in the form $y = ab^x$, where $a \neq 0$, $b > 0$ and $b \neq 1$.
- Property of Equality for Exponential Functions: If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.
- Property of Inequality for Exponential Functions: If $b > 1$, then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.

Logarithms and Logarithmic Functions

(Lessons 9-2 through 9-4)

- Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that $\log_b x = y$ if and only if $b^y = x$.
- The logarithm of a product is the sum of the logarithms of its factors.
- The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.
- The logarithm of a power is the product of the logarithm and the exponent.
- The Change of Base Formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Natural Logarithms (Lesson 9-5)

- Since the natural base function and the natural logarithmic function are inverses, these two can be used to “undo” each other.

Exponential Growth and Decay (Lesson 9-6)

- Exponential decay: $y = a(1 - r)^t$ or $y = ae^{-kt}$
- Exponential growth: $y = a(1 + r)^t$ or $y = ae^{kt}$

Key Vocabulary

common logarithm (p. 528)
exponential decay (p. 500)
exponential equation (p. 501)
exponential function (p. 499)
exponential growth (p. 500)
exponential inequality (p. 502)
logarithm (p. 510)
logarithmic equation (p. 512)
logarithmic function (p. 511)
logarithmic inequality (p. 512)
natural base, e (p. 536)
natural base exponential function (p. 536)
natural logarithm (p. 537)
natural logarithmic function (p. 537)
rate of decay (p. 544)
rate of growth (p. 546)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined word(s) to make a true statement.

1. In $x = b^y$, y is called the <u>logarithm.</u>
2. The change in the number of bacteria in a Petri dish over time is an example of <u>exponential decay.</u>
3. The <u>natural logarithm</u> is the inverse of the exponential function with base 10.
4. The irrational number 2.71828... is referred to as the <u>natural base, e.</u>
5. If a savings account yields 2% interest per year, then 2% is the <u>rate of growth</u>.
6. Radioactive half-life is used to describe the <u>exponential decay</u> of a sample.
7. The inverse of an exponential function is a <u>composite function</u>.
8. If $24^{2y+3} = 24^{y-4}$, then $2y + 3 = y - 4$ by the <u>Property of Equality for Exponential Functions.</u>
9. The <u>Power Property of Logarithms</u> shows that $\ln 9 < \ln 81$.

Lesson-by-Lesson Review

9-1 Exponential Functions (pp. 498–506)

Determine whether each function represents exponential *growth* or *decay*.

10. $y = 5(0.7)^x$ **11.** $y = \frac{1}{3}(4)^x$

Write an exponential function for the graph that passes through the given points.

12. (0, −2) and (3, −54)

13. (0, 7) and (1, 1.4)

Solve each equation or inequality. Check your solution.

14. $9^x = \frac{1}{81}$ **15.** $2^{6x} = 4^{5x+2}$

16. $49^{3p+1} = 7^{2p-5}$ **17.** $9^{x^2} \le 27^{x^2-2}$

18. POPULATION The population of mice in a particular area is growing exponentially. On January 1, there were 50 mice, and by June 1, there were 200 mice. Write an exponential function of the form $y = ab^x$ that could be used to model the mouse population y of the area. Write the function in terms of x, the number of months since January.

Example 1 Write an exponential function for the graph that passes through (0, 2) and (1, 16).

$y = ab^x$	Exponential equation
$2 = ab^0$	Substitute (0, 2) into the exponential equation.
$2 = a$	Simplify.
$y = 2b^x$	Intermediate function
$16 = 2b^1$	Substitute (1, 16) into the intermediate function.
$8 = b$	Simplify.
$y = 2(8)^x$	

Example 2 Solve $64 = 2^{3n+1}$ for n.

$64 = 2^{3n+1}$	Original equation
$2^6 = 2^{3n+1}$	Rewrite 64 as 2^6 so each side has the same base.
$6 = 3n + 1$	Property of Equality for Exponential Functions
$\frac{5}{3} = n$	The solution is $\frac{5}{3}$.

9-2 Logarithms and Logarithmic Functions (pp. 509–517)

(continued on the next page)

Write each equation in logarithmic form.

19. $7^3 = 343$ **20.** $5^{-2} = \frac{1}{25}$

Write each equation in exponential form.

21. $\log_4 64 = 3$ **22.** $\log_8 2 = \frac{1}{3}$

Evaluate each expression.

23. $4^{\log_4 9}$ **24.** $\log_7 7^{-5}$

25. $\log_{81} 3$ **26.** $\log_{13} 169$

Example 3 Solve $\log_9 n > \frac{3}{2}$.

$\log_9 n > \frac{3}{2}$	Original inequality
$n > 9^{\frac{3}{2}}$	Logarithmic to exponential inequality
$n > (3^2)^{\frac{3}{2}}$	$9 = 3^2$
$n > 3^3$	Power of a Power
$n > 27$	Simplify.

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9-2 Logarithms and Logarithmic Functions (pp. 509–517)

Solve each equation or inequality.

27. $\log_4 x = \frac{1}{2}$

28. $\log_{81} 729 = x$

29. $\log_8 (x^2 + x) = \log_8 12$

30. $\log_8 (3y - 1) < \log_8 (y + 5)$

31. **CHEMISTRY** $\text{pH} = -\log(H^+)$, where H^+ is the hydrogen ion concentration of the substance. How many times as great is the acidity of orange juice with a pH of 3 as battery acid with a pH of 0?

Example 4 **Solve $\log_3 12 = \log_3 2x$.**

$\log_3 12 = \log_3 2x$	Original equation
$12 = 2x$	Property of Equality for Logarithmic Functions
$6 = x$	Divide each side by 2.

9-3 Properties of Logarithms (pp. 520–526)

Use $\log_9 7 \approx 0.8856$ and $\log_9 4 \approx 0.6309$ to approximate the value of each expression.

32. $\log_9 28$

33. $\log_9 49$

34. $\log_9 144$

35. $\log_9 63$

Solve each equation. Check your solutions.

36. $\log_5 7 + \frac{1}{2}\log_5 4 = \log_5 x$

37. $2\log_2 x - \log_2 (x + 3) = 2$

38. $\log_6 48 - \log_6 \frac{16}{5} + \log_6 5 = \log_6 5x$

39. **SOUND** Use the formula $L = 10 \log_{10} R$, where L is the loudness of a sound and R is the sound's relative intensity, to find out how much louder 10 alarm clocks would be than one alarm clock. Suppose the sound of one alarm clock is 80 decibels.

Example 5 **Use $\log_{12} 9 \approx 0.884$ and $\log_{12} 18 \approx 1.163$ to approximate the value of $\log_{12} 2$.**

$\log_{12} 2 = \log_{12} \frac{18}{9}$	Replace 2 with $\frac{18}{9}$.
$= \log_{12} 18 - \log_{12} 9$	Quotient Property
$\approx 1.163 - 0.884$ or 0.279	

Example 6
Solve $\log_3 4 + \log_3 x = 2\log_3 6$.

$\log_3 4 + \log_3 x = 2\log_3 6$	
$\log_3 4x = 2\log_3 6$	Product Property of Logarithms
$\log_3 4x = \log_3 6^2$	Power Property of Logarithms
$4x = 36$	Property of Equality for Logarithmic Functions
$x = 9$	Divide each side by 4.

Mixed Problem Solving
For mixed problem-solving practice, see page 934.

9-4 Common Logarithms (pp. 528–533)

Solve each equation or inequality. Round to four decimal places.

40. $2^x = 53$ **41.** $2.3^{x^2} = 66.6$

42. $3^{4x-7} < 4^{2x+3}$ **43.** $6^{3y} = 8^{y-1}$

44. $12^{x-5} \geq 9.32$ **45.** $2.1^{x-5} = 9.32$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

46. $\log_4 11$ **47.** $\log_2 15$

48. MONEY Diane deposited \$500 into a bank account that pays an annual interest rate r of 3% compounded quarterly. Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to find how long it will take for Diane's money to double.

Example 7 Solve $5^x = 7$.

$5^x = 7$	Original equation
$\log 5^x = \log 7$	Property of Equality for Logarithmic Functions
$x \log 5 = \log 7$	Power Property of Logarithms
$x = \frac{\log 7}{\log 5}$	Divide each side by log 5.
$x \approx \frac{0.8451}{0.6990}$ or 1.2090	Use a calculator.

9-5 Base *e* and Natural Logarithms (pp. 536–542)

Write an equivalent exponential or logarithmic equation.

49. $e^x = 6$ **50.** $\ln 7.4 = x$

Solve each equation or inequality.

51. $2e^x - 4 = 1$ **52.** $e^x > 3.2$

53. $-4e^{2x} + 15 = 7$ **54.** $\ln 3x \leq 5$

55. $\ln (x - 10) = 0.5$ **56.** $\ln x + \ln 4x = 10$

57. MONEY If you deposit \$1200 in an account paying 4.7% interest compounded continuously, how long will it take for your money to triple?

Example 8 Solve $\ln (x + 4) > 5$.

$\ln (x + 4) > 5$	Original inequality
$e^{\ln (x+4)} > e^5$	Write each side using exponents and base e.
$x + 4 > e^5$	Inverse Property of Exponents and Logarithms
$x > e^5 - 4$	Subtract 4 from each side.
$x > 144.4132$	Use a calculator.

9-6 Exponential Growth and Decay (pp. 544–550)

58. BUSINESS Able Industries bought a fax machine for \$250. It is expected to depreciate at a rate of 25% per year. What will be the value of the fax machine in 3 years?

59. BIOLOGY For a certain strain of bacteria, k is 0.872 when t is measured in days. Using the formula $y = ae^{kt}$, how long will it take 9 bacteria to increase to 738 bacteria?

60. CHEMISTRY Radium-226 has a half-life of 1800 years. Find the constant k in the decay formula for this compound.

61. POPULATION The population of a city 10 years ago was 45,600. Since then, the population has increased at a steady rate each year. If the population is currently 64,800, find the annual rate of growth for this city.

Example 9 **A certain culture of bacteria will grow from 500 to 4000 bacteria in 1.5 hours. Find the constant k for the growth formula. Use $y = ae^{kt}$.**

$y = ae^{kt}$	Exponential growth formula
$4000 = 500\, e^{k(1.5)}$	Replace y with 4000, a with 500, and t with 1.5.
$8 = e^{1.5k}$	Divide each side by 500.
$\ln 8 = \ln e^{1.5k}$	Property of Equality for Logarithmic Functions
$\ln 8 = 1.5k$	Inverse Property of Exponents and Logarithms
$\frac{\ln 8}{1.5} = k$	Divide each side by 1.5.
$1.3863 \approx k$	Use a calculator.

The constant k for this type of bacteria is about 1.3863.